Multiple Choice: Indicate your answer in the box to the right of each question.

1. If $a = 0.5$, $b = \frac{1}{3}$, and $c = 24$, find the value of $a^4bc^2 - ca^2$

(a) -12 (b) -6 (c) 6 (d) 12 (e) 18

1.

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2. $\tan A = \frac{20\sqrt{17}}{17\sqrt{20}}$. Which of these expressions is equal to $\sin A$?

(a) $\sqrt{\frac{20}{17}}$ (b) $\sqrt{\frac{20}{37}}$ (c) $\sqrt{\frac{17}{37}}$ (d) $\sqrt{\frac{17}{20}}$ (e) $\sqrt{\frac{3}{37}}$

2.

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3. Consider all the numbers that can be expressed as sums of distinct powers of 3:
1, 3, 4, 9, 10, 12, 13, 27, … What is the 20th number in this increasing sequence?

(a) 90 (b) 91 (c) 93 (d) 94 (e) 99

3.

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4. The quadratic equations $y = ax^2 + 5x + 5$ and $y = ax^2 - 3x - 11$ have one root in common. What is it?

(a) a (b) -5 (c) -2 (d) 5 (e) 9

4.

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5. If $\log_2(\log_2(\log_2 x)) = 2$, how many digits long is the decimal representation of x?

(a) 4 (b) 5 (c) 6 (d) 15 (e) 16

5.

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6. What is the sum of the squares of the roots of $y = x^2 + 4x - 6$?

(a) -8 (b) -6 (c) -4 (d) 24 (e) 28

6.

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7. Compute $\sqrt{1 + (49)(50)(51)(52)}$

(a) 50 (b) 51 (c) 2549 (d) 2551 (e) 2601

7.

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8. How many terms of the arithmetic sequence 75, 122, 169, 216, … are less than 2017?

(a) 41 (b) 42 (c) 43 (d) 44 (e) 45

8.

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9. Three circle chords of lengths 6, 8, and 10 form a triangle. Find the distance between the midpoints of the minor arcs determined by the two shorter chords.

(a) $5\sqrt{2}$ (b) $4\sqrt{5}$ (c) 5 (d) $5\sqrt{6}$ (e) 10

9.

Short Answer: Write your answer and show your work in the space below each question.
Clearly indicate your final answer by drawing a $\boxed{\text{box}}$ around it.

10. Simplify the expression: $\left(\frac{2x}{6x^2-5x+1}\right)\left(\frac{2x^2+5x-3}{7x^2+21x}\right)$

11. Solve for x: $\left(\frac{1}{2}\right)^x = 8^{2x+7}$

12. What is the biggest multiple of 12 whose digits are all different?

13. Two circles of radii 1 and 4 are externally tangent. A line is drawn tangent to both circles (at different points). Compute the distance between the points of tangency of the line to the two circles.

14. If 20^n is the largest power of 20 that is a factor of $2017!$, compute n.

15. Triangle ABC has sides 5, 12, and 13, while triangle ABD has sides 9, 12, and 15. The two triangles overlap, so that $AB = 12$ and C is on segment $\overline{AD}$. Find the distance from C to $\overline{BD}$.

16. You have 2017 identical looking coins. They are indistinguishable except for one counterfeit coin which is slightly heavier than the others. The weight difference is subtle enough that it takes a weighing on a scale to notice it. Unfortunately you only have a balance scale that can compare two equal stacks of coins and determine which is heavier (so if you weigh 100 fair coins vs 99 fair and 1 counterfeit coin, the side with the counterfeit will be heavier). You can guarantee to find the fake coin in at most X uses of the scale. Find X.

Long Answer: Write your solution in the space below each question. Make sure you include sufficient justification.

17. The Stern-Brocot sequence can be formed as follows: Let $s_1 = \{0,1\}$. We keep forming the next sequence by inserting between each neighbors the value of their sum. So $s_2 = \{0,1,1\}$, $s_3 = \{0,1,1,2,1\}$, etc.

a. Find s_5
b. State the formula for the number of terms in s_n
c. One of the terms of s_n is the Fibonacci number F_n. For example, s_4 contains 3 and s_5 contains 5. Prove this.
d. The sum of the terms in s_n is $\frac{3^{n-1}+1}{2}$. Prove this result.

18. The chess knight is a piece that moves from a square to another square whose center is exactly $\sqrt{5}$ units away. For example, on the right, the white knight Na can move to the squares x, and the black knight Nb can move to the squares y.
If we only allow moves to empty squares show that no sequence of moves can turn the first of the below positions into the second.

Na		y
Nb		x
	x	y

Na		Na
Nb		Nb

Na		Nb
Nb		Na